Unifying gravity and electromagnetism using analogies based on electromagnetism for gravity

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Abstract

Gravity and electromagnetism are unified at the level of a 4-potential by re-examining the most obvious choice at the Lagrangian level:

$$\mathcal{L}_{GEM} = -\frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla^\mu A_\mu \nabla_\mu A^\mu.$$

The GEM Lagrangian has mass and electric current densities coupled to the 4-potential with different signs: an attractive one for gravity and one where like charges repel for electromagnetism. The reducible asymmetric tensor $\nabla^\mu A_\mu$ can be represented by two irreducible field strength tensors: a symmetric tensor for gravity and an antisymmetric electromagnetic field strength tensor $F^{\mu\nu}$ for electromagnetism. Any long-range interactions based on this Lagrangian will require even and odd integer-spin fields due to the presence of these two irreducible field strength tensors.

The Hamiltonian density results from a direct calculation. The energy density of the stress tensor remains defined and non-zero even if the connection is zero, a technical advantage over general relativity.

By varying the action with respect to the potential, these four-dimensional wave equations result:

$$J_q^\mu - J_m^\mu = \nabla_\nu \nabla^\nu A^\mu.$$

If the mass current density J_m^μ is zero, one sees the Maxwell equations. If the electric charge density ρ_q is zero but the mass density ρ_m is not, in the static case, Newton's field equation for gravity results. For the non-static case, the equation for gravity transforms like the first component of a 4-vector under a Lorentz transformation. The metric is fixed because the 4-potential is varied, not the metric field, but fixed in no way implies flat. The unified field equations constrains the metric in a way that the Maxwell equations do not.

An inverse squared distance function solves a four-dimensional wave equation. This appears to create a significant problem, since the derivative of the potential yields an inverse cubed force law which is not physical. The static $1/R$ potential function for Newton's static field equation is physically relevant. If the system is no longer static, but only barely dynamic, then perturbation theory is required. A normalized, linear perturbation potential function was found whose derivative has the correct inverse squared distance dependence needed for a classical force.

An electrically-neutral, normalized, linear perturbation 4-potential is used in a gravitational Lorentz 4-force equation. The solution to the force equation, with a flat spacetime metric boundary condition, yields the following dynamic metric equation:

$$(d\tau)^2 = e^{-2\frac{GM}{c^2 R}}(dt)^2 - e^{2\frac{GM}{c^2 R}}(dR/c)^2$$

This exponential metric equation has the same ten parameterized post-Newtonian (PPN) parameters as general relativity, so it will pass all the same tests of the equivalence principle and the Schwarzschild metric. The second-order PPN parameters for the two metrics differ by 15%, so the model could be confirmed or rejected experimentally. Current strong field test such as energy loss by gravitational waves which probe to 2.5 PPN order should be consistent with the exponential metric via a small adjustment to the model of the masses in the system.

A path from the gravitational Lorentz 4-force to the Newtonian classical 3-force is established. In the derivation, there is a choice between two derivatives as a consequence of applying the chain rule to the 4-momentum when taking its derivative. A complete analysis will have two effects, the standard mass times acceleration and a new classical gravitational effect which dictates where inertial mass traveling at the same velocity is distributed in space. This may lead to new explanations for the rotation profile of thin galaxies without requiring dark matter or a modification of Newtonian mechanics (MOND). A new approach to the horizon and flatness problems of early big bang cosmology is possible. Both these application of the new classical gravitational effect on inertial mass distribution will require mathematical modeling to see how well it fits the data.

A preliminary investigation into quantizing the radiation modes for the field theory is conducted. The process will be very similar to the Gupta-Bleuler method of fixing the Lorenz gauge due to the similarities between the field equations. The key difference is that the unified field equations have two spin fields, one even, the other odd. The scalar polarization mode of emission for a spin 2 field will not have the issue of a negative norm that plagues a spin 1 field.

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I. INTRODUCTION

The goal of this paper is to create one mathematical structure for gravity and electromagnetism that can be quantized. The difference between gravity and electromagnetism is the oldest core problem facing physics, going back to studies of electromagnetism in the eighteenth century. Gravity was the first inverse square law, discovered by Isaac Newton. After twenty years of effort, he was able to show that inside a hollow massive shell, the gravitational field would be zero. Ben Franklin, in his studies of electricity, demonstrated a similar property for an electrically charged ball inside an insulated conducting can. Joseph Priestly realized this meant that the electrostatic force was governed by an inverse square law like gravity. Due to his experimental work, Coulomb got the credit for the electrostatic force law modeled on Newton's law of gravity.[5]

Over a hundred years later, Einstein started from the tensor formalism of electromagnetism on the road to general relativity. Instead of an antisymmetric field strength tensor, Einstein used a symmetric tensor because the metric tensor is symmetric. There is a precedence for transforming mathematical structures between gravity and electromagnetism.

Previous efforts to use 4-potentials report failures due in part to considering analogies too close to electromagnetism. Although free to choose the sign of a mass current, researchers chose the same sign used for electromagnetism where like charges repel.[3, 12] Other efforts created an attractive gravitational force, but forgot Einstein's insight that a symmetric tensor was required.[10] Perhaps the greatest barrier however was the inverse squared potential that solved the field equations suggest a non-physical inverse-cubed force law. Perturbation theory near a physically relevant potential will be needed to generate the correct force law.

In this paper, analogies to electromagnetism are used for gravity, respecting differences. The mass current always has an opposite sign to electric current, and the second-rank gravitational source tensor is symmetric. An inverse distance potential is physically relevant, so a normalized 4-potential near that classical result will be the focus. By staying close to electromagnetism, particularly by working with a linear field theory, the difficulty of quantizing gravity may dissolve.

II. LAGRANGE DENSITIES

The classic electromagnetic Lagrangian density has two terms: one for electric charge density coupling to the potential and another for the antisymmetric second-rank field strength tensor $F^{\mu\nu}$:

$$\mathcal{L}_{EM} = -\frac{1}{c} J_q^\mu A_\mu - \frac{1}{4c^2}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{1}$$

An analogous Lagrangian for gravity should contain these components, but changes are required. Gravity would couple to mass current density, not electric current density or a rank 2 energy density tensor. Mass does not have the same units as electric charge, so mass will have to be multiplied by the square root of Newton's gravitational constant G to keep the units identical. Where there is a negative electric current density, a positive mass current density will be substituted. The change in sign is required so that like mass currents attract for gravity. Because gravity effects metrics which are symmetric, the source of gravity must also be symmetric. The minus sign within the electromagnetic field strength tensor $F^{\mu\nu}$ antisymmetric will be made positive. In order that the symmetric object transforms like a tensor, the standard derivative must be replaced by a covariant derivative:

$$\mathcal{L}_G = +\frac{1}{c} J_m^\mu A_\mu - \frac{1}{4c^2}(\nabla^\mu A_\mu + \nabla^\nu A_\nu)(\nabla_\mu A^\nu + \nabla_\nu A^\mu). \tag{2}$$

The unified Lagrangian will be the sum of these two, $\mathcal{L}_{\mathcal{EM}}$ and $\mathcal{L}_{\mathcal{G}}$, which separately only apply if the other force is not in effect. Without loss of generality, the regular derivatives in the electromagnetic Lagrangian (Eq. 1) can be written as covariant derivatives. This leads to the unified Lagrangian for gravity and electromagnetism:

$$\mathcal{L}_{GEM} = -\frac{1}{c}(J_q^\mu - J_m^\mu) A_\mu - \frac{1}{2c^2}\nabla^\mu A_\nu \nabla_\mu A^\nu. \tag{3}$$

The Fermi Lagrangian of electromagnetism is a subset of Eq. 3. This establishes a link to electromagnetism. Because a dynamic metric is part of the Lagrangian, the GEM Lagrangian could impose constraints on the dynamics of the metric, which is a central accomplishment of general relativity. The possibility to do both gravity and electromagnetism is here.

Because the gauge was not fixed, there is more freedom for this Lagrangian, which is required if this field equation is to do gravity in addition to electromagnetism.

III. UNIFIED FIELD EQUATIONS

Write out the action based on the GEM Lagrange density (Eq. 3):

$$S = \int d^4x \sqrt{-det(g)}(\frac{1}{c}(J_q^\mu - J_m^\mu)A_\mu - \frac{1}{2c^2}\nabla^\mu A_\nu \nabla_\mu A^\nu) \tag{4}$$

Varying the action by the vector field A^μ to generate the field equations:

$$J_q^\mu - J_m^\mu = \nabla_\nu \nabla^\nu A^\mu. \tag{5}$$

This is a four-dimensional wave equation with two sources, one for electricity, the other for gravity. The Maxwell equations are apparent in the physical situation where the mass current density J_m^μ is zero. Newton's field equation for gravity is the static field equation when there is no electric current density ρ_q. The field equations are covariant under a Lorentz transformation, behaving like a 4-vector. One of the justifications for general relativity - to make Newton's field equation for gravity covariant under a Lorentz transformation - is no longer compelling because Eq. 5 is manifestly covariant.[9]

Because the metric was not varied, that indicates that it must be fixed. With the Maxwell equations, there are no constraints on the metric, so a metric must be supplied as part of the background structure for the theory. With the GEM field equations, the derivatives of the metric are part of the field equations. The process of finding solutions to the field equations will necessitate constraints on the derivative of the metric. Thus the metric should not be specified beforehand. The rest of the background structure for the Maxwell equations - things like the 4-dimensional structure, the differential manifold - will still be necessary for the GEM field equations.

The classical fields will be expressed in terms of covariant derivatives of the potential. The symmetric and antisymmetric field strength tensors are very similar, differing only in the sign of the tensor $\nabla^\mu A^\nu$. The electric field $\overrightarrow{E}$ and magnetic field $\overrightarrow{B}$ together represent all the information in the antisymmetric field strength tensor. Symmetric analogs $\overrightarrow{e}$ and $\overrightarrow{b}$ will be defined to do a similar job for the symmetric field strength tensor. In addition, there is a four component field for the terms along the diagonal which will be noted as g^μ. To make a connection to the classical fields of gravity and electromagnetism, use the following mappings.

$$E^i = \partial A^i - \partial^i A^0 \tag{6}$$

$$e^i = \partial^0 A^i - \partial^i A^0 - 2\Gamma^{0i}_{\nu} A^\nu \tag{7}$$

$$\epsilon^{ijk} B^k = \partial^i A^j - \partial^j A^i \tag{8}$$

$$b^k = -\partial^i A^j - \partial^j A^i - 2\Gamma^{ij}_{\nu} A^\nu \qquad i \neq j \neq k \tag{9}$$

$$g^\mu = \partial^\mu A^\mu - \Gamma^{\mu\mu}_{\nu} A^\nu. \tag{10}$$

The set of five classical fields will transform like a rank 2 asymmetric tensor. Under a Lorentz transformation, the three gravity fields will mix, as will the two electromagnetic fields, but there will be no mixing between gravity and electromagnetism since the fields are in separate, irreducible tensors.

The first row and column of the asymmetric field strength tensor $\nabla^\nu A^\mu$ is the sum of the electric field $\overrightarrow{E}$ and the symmetric analog $\overrightarrow{e}$. The rest of the off-diagonal terms are the sum of the magnetic field $\overrightarrow{B}$ and its symmetric counterpart $\overrightarrow{b}$. The diagonal of the field strength tensor is g^μ. If the sum of the components of g^μ is the trace of the asymmetric field strength tensor, so is an invariant. The trace contains information about both the gauge and the Christoffel symbol coefficients. The ability to choose an arbitrary gauge for electromagnetism is equivalent to ignoring constraints imposed by gravity, which is common for nearly all practical problems with electromagnetism.

Substitute the classical fields (Eq. 6-10) into the field equations (Eq. 5), starting with the first field equation:

$$\begin{aligned}\rho_q - \rho_m &= c\nabla_\mu \nabla^\mu \phi \\ &= \frac{c}{2}(\overrightarrow{\nabla} \cdot \overrightarrow{E} + \overrightarrow{\nabla} \cdot \overrightarrow{e}) + \frac{\partial g^0}{\partial t}.\end{aligned} \tag{11}$$

This has Gauss' law and analogous equation for gravity. The Newtonian gravitational field equation is embedded here under the following conditions:

$$\rho_m = c\nabla^2 \phi = -c\overrightarrow{\nabla} \cdot \overrightarrow{e} \tag{12}$$

$$iff \qquad \rho_q = 0,\ \overrightarrow{\nabla} \cdot \overrightarrow{E} = 0,\ and\ \frac{\partial g^0}{\partial t} = 0. \tag{13}$$

If the time derivative of g^0 is not zero, then the field equations for gravity incorporate time directly. This is significant because one justification for general relativity is to make the form of Newton's field equations dynamic.[10, chapter 7]

With a particular choice of reference frame such that all symmetric derivatives of the potential vanish, the relativistic form of Eq. 13 concerns changes in the Christoffel symbol components exclusively:

$$\rho_m = -\partial_\mu \Gamma_\nu^{0\mu} A^\nu \tag{14}$$

The second order change in the metric is determined by the mass charge density. It is significant that the Riemann curvature tensor is not being used here because this unification model is fundamentally distinct from general relativity.

Gauss' law can be isolated under different physical conditions:

$$\begin{aligned} \rho_q &= \frac{1}{c}\frac{\partial^2 \phi}{\partial t^2} - c\nabla^2 \phi = c\overrightarrow{\nabla} \cdot \overrightarrow{E} \\ iff \qquad & \rho_q = 0,\ \overrightarrow{\nabla} \cdot \overrightarrow{e} = 0\ and\ \frac{\partial g^0}{\partial t} = 0. \end{aligned} \tag{15}$$

To make the discussion of classical unification more concrete, imagine a proton at the center of a 1 cm sphere. The electric charge density would be $3.82x10^{-14}C/m^3$. The mass charge density would be $3.25x10^{-27}C/m^3$, thirteen orders of magnitude smaller than the electric charge density. This model does not address why there is such a significant difference. The unified charge density for a proton is slightly less than the electric charge density, although not measurably so since the electric charge is not known to thirteen significant digits. If a proton approaches this sphere, there will be a repulsive electric force and an attractive gravitational force, combining for a net unified charge of slightly less than the full electric charge density. For an electron in the sphere, the unified charge density would be slightly greater because for a proton approaching the sphere, both electrical and gravitational forces are attractive.

Repeat the exercise for the 3-vector field equation.

$$\begin{aligned} \overrightarrow{J}_q - \overrightarrow{J}_m &= \frac{1}{c}\frac{\partial^2 \overrightarrow{A}}{\partial t^2} - c\nabla^2 \overrightarrow{A} \\ &= \frac{1}{2}\left(-\frac{\partial \overrightarrow{E}}{\partial t} + c\overrightarrow{\nabla} \times \overrightarrow{B} + \frac{\partial \overrightarrow{e}}{\partial t} + c\overrightarrow{\nabla} \boxtimes \overrightarrow{b}\right) + c\overrightarrow{\nabla} g^u. \end{aligned} \tag{16}$$

$$where \quad (\overrightarrow{a} \boxtimes \overrightarrow{b})^k = -a^i b^j - a^j b^i \quad i \neq j \neq k$$

This has Ampere's law and a symmetric analog for gravity.

The model for classical gravitational and electromagnetic field equations is expressed with 4-vectors, tensors of rank one. Einstein's field equations use tensors of rank two. Therefore the two approaches are fundamentally different. Although the field equations are rank one, the field strength tensor is second rank, consistent with arguments that a second-rank field strength tensor is required to characterize a dynamic metric. Second-order derivatives of the metric arise from the divergence of the Christoffel symbols as seen in Eq. 14. At no point is there a need for the Ricci curvature tensor.

With no gravitational fields or mass currents, the Maxwell source equations result. The homogeneous Maxwell equations are vector identities, unaffected by unification.

IV. THE HAMILTONIAN DENSITY AND STRESS TENSOR

The conical momentum density conjugate to the potential A^μ is simple because of the quadratic nature of the GEM Lagrangian (Eq. 3):

$$\pi^\mu = -\frac{\partial A^\mu}{\partial t} \tag{17}$$

The free field ($J_q^\mu = J_m^\mu = 0$) Hamiltonian density can be written with the classical fields:

$$H = -\frac{1}{2}g_0^2 + \frac{1}{2}\overrightarrow{g}^{\,2} - \frac{1}{2}\overrightarrow{E}\,\overrightarrow{e} + \frac{1}{4}\overrightarrow{B}^{\,2} + \frac{1}{4}\overrightarrow{b}^{\,2}). \tag{18}$$

Four of the five terms are quadratics of the fields as one might expect. The most unusual term is the product of the electric field $\overrightarrow{E}$ and its analog, $\overrightarrow{e}$. In flat spacetime where $\overrightarrow{\nabla}\phi = 0$, the two fields have opposite signs, $\overrightarrow{E} = -\overrightarrow{e}$, so the term will be a positive quadratic of the field. It is unclear all the implications for different physical situations.

The stress tensor $T^{\mu\nu}$ is a generalization of the Hamiltonian density. To conserve angular momentum, the tensor must be symmetric under the exchange of indices. Fortunately that is the case for the GEM Lagrangian without any modifications. The energy density term T^{00} is the Hamiltonian, Eq. 18. The rest of the momentum terms can be written with the covariant classical fields:

$$T^{0i} = \frac{1}{2}g^0(\overrightarrow{E} + \overrightarrow{e}) - \frac{1}{2}\overrightarrow{g}(\overrightarrow{E} - \overrightarrow{e}) + \frac{1}{4}(\overrightarrow{E} - \overrightarrow{e}) \times \overrightarrow{B} + \frac{1}{4}(\overrightarrow{E} - \overrightarrow{e}) \boxtimes \overrightarrow{b} \tag{19}$$

$$\begin{aligned} T^{ij} = & -\frac{1}{4}(\overrightarrow{E} + \overrightarrow{e})_i(\overrightarrow{E} + \overrightarrow{e})_j - \frac{1}{4}(\overrightarrow{b} + (-1^{i+j})\overrightarrow{B})_i(\overrightarrow{b} - (-1^{i+j})\overrightarrow{B})_j \\ & -\frac{1}{2}g_i(\overrightarrow{b} + (-1^{i+j})\overrightarrow{B})_k - \frac{1}{4}g_j(\overrightarrow{b} - (-1^{i+j})\overrightarrow{B})_k \end{aligned} \tag{20}$$

If there are no gravitational fields, the expected cross product between the electric and magnetic fields results for the momentum density. The energy and momentum densities are difficult to define in general relativity because choosing coordinates where the connection is zero can eliminate the energy density. In the GEM model, the three gravitational fields depend on both the connection and the potential. As such, even if the connection is zero, the derivative of the potential may not be.

The stress tensor leads to the conservation laws, $\partial_\nu T^{\mu\nu} = 0$, using the definition of the stress energy tensor and the Euler-Lagrange equation. If the system is isolated, integrating $\partial_\nu T^{\mu\nu}$ over all of 3-space leads to the total energy and momentum conservation laws for the field.

V. GEM FIELD EQUATION SOLUTIONS

The task is to find a physically relevant solution to the GEM field equations, Eq. 5. The Poisson field equation of classical Newtonian gravity can be solved by an inverse distance potential, $\frac{1}{R}$. The potential has a point singularity where $R = 0$. The GEM field equations are relativistic, so time must be incorporated. An inverse distance potential does not solve the field equations in four dimensions. The potential $A_\mu = (\frac{1}{\sigma^2}, \overrightarrow{0})$ solves the field equations, where σ^2 is the Lorentz invariant distance squared, the negative of the Lorentz invariant interval squared, $-(c\tau)^2$. Distance is used instead of the interval because classical gravity depends on distance, not time. The potential has as a singularity that is the entire lightcone, where $\sigma^2 = 0$. The potential is not relevant in the classical domain since its derivative will not be an inverse square as required for classical forces.

Gravity is a weak effect. It is common in quantum mechanics to normalize a potential and study linear perturbations of weak fields, an approach that will be followed. Assume spherical symmetry. Form a normalized potential with a linear perturbation:

$$A^{\mu} = (\frac{\sqrt{G}h}{c^2\sigma^2}, \overrightarrow{0}) \rightarrow (\frac{c/\sqrt{G}}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}R)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2}, \overrightarrow{0}). \tag{21}$$

Take the derivative with respect to t and R, keeping only terms to first order in the spring constant k:

$$\begin{aligned}
\frac{\partial \phi}{\partial t} &= \frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2} + O(k^2) \\
\nabla \phi &= -\frac{c^2}{\sqrt{G}} \frac{k}{\sigma^2} + O(k^2).
\end{aligned} \tag{22}$$

The change in the potential is a function of a spring constant k over sigma squared. The classical Newtonian dependence on distance is an inverse square, so this is promising. A potential that applies exclusively to gravity is sought, yet the non-zero gradient of ϕ indicates an electric field. The sign of the spring constant k does not effect the solution to the four dimensional wave field equations but does change the derivative of the potential. A potential that only has derivatives along the diagonal of the field strength tensor $A^{\mu,\nu}$ can be constructed from two potentials that differ by spring constants that either constructively interfere to create non-zero derivatives, or destructively interfere to eliminate derivatives. With this in mind, construct a potential that will be electrically neutral:

$$\begin{aligned}
A^{\mu} &= \frac{c}{\sqrt{G}} (\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2} \\
&+ \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2}, \\
&\quad \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2} \\
&+ \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}t)^2}, \\
&\quad \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2} \\
&+ \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}t)^2}, \\
&\quad \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}t)^2} \\
&+ \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}x)^2 + (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}y)^2 + (\frac{1}{\sqrt{2}} + \frac{k}{\sigma^2}z)^2 - (\frac{1}{\sqrt{2}} - \frac{k}{\sigma^2}t)^2}).
\end{aligned} \tag{23}$$

Take the contravariant derivative of this potential, keeping only the terms to first order in the spring constant k. The contravariant derivative flips the sign for the 3-vector.

$$\partial^\nu A^\mu = \frac{c^2}{\sqrt{G}}\frac{k}{\sigma^2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + O(k^2) \tag{24}$$

All this work was required to get a multiple of the identity matrix!

VI. RELATIVISTIC GRAVITATIONAL FORCE

The electromagnetic Lorentz force density arises from moving charges in an electromagnetic field:

$$F^\mu_{EM} = J_{q\,\nu}(\partial^\mu A^\nu - \partial^\nu A^\mu) = \frac{d\rho_m U^\mu}{d\tau}. \tag{25}$$

If the sign of the electric charge q where changed, this would change the sign of the 4-force, so there are two distinguishable signs for electric charge. Like electric charges repel because the force has a positive sign.

An analogous gravitational Lorentz force can be created with precisely the same substitutions as were used for the Lagrange density:

$$F^\mu_G = -J_{m\,\nu}(\nabla^\mu A^\nu + \nabla^\nu A^\mu) = \frac{d\rho_m U^\mu}{d\tau}. \tag{26}$$

If the sign of the mass current were changed, then the sign of the inertial mass density in the force would also need to change, resulting in no change, so there is only one distinguishable sign for mass charge. Like mass charges attract because the force has a negative sign.

Plug the weak-field gravitational approximation (Eq. 24) into the gravitational Lorentz 4-force equation (Eq. 26), rewriting the mass current density as the product of mass charge density times velocity:

$$F^\mu_G = \rho_m(-\frac{ck}{\sigma^2}U^0, \frac{ck}{\sigma^2}\overrightarrow{U}) = \rho_m(\frac{dU^0}{d\tau}, \frac{d\overrightarrow{U}}{d\tau}) \tag{27}$$

This is the weak-field gravitational Lorentz 4-force density. These are first-order differential equations. The weak equivalence principle equates inertial and passive gravitational mass densities. This principle will be assumed, but is supported by experiments.[15] So that the equations have the same variable, the squared Lorentz invariant distance σ^2 needs to be substituted for by the squared interval $-c^2\tau^2$:

$$\frac{dU^0}{d\tau} - \frac{k}{c\tau^2}U^0 = 0 \tag{28}$$

$$\frac{d\overrightarrow{U}}{d\tau} + \frac{k}{c\tau^2}\overrightarrow{U} = 0. \tag{29}$$

Solve for the velocity, U^μ. The solution involves exponentials:

$$U^\mu = (ve^{-\frac{k}{\tau}}, \overrightarrow{V}e^{\frac{k}{\tau}}). \tag{30}$$

For flat spacetime, $U^\mu = (v, \overrightarrow{V})$. The constraint on relativistic velocities in flat spacetime is:

$$U^\mu U_\mu = \frac{(cdt)^2 - dR^2}{d\tau^2} = c^2 = v^2 - \overrightarrow{V} \cdot \overrightarrow{V}. \tag{31}$$

This will be the case for the velocity solution should the spring constant be zero or the interval infinite. Substitute the solution, Eq. 30, into the constraint, Eq. 31, multiplying through by $d\tau^2$.

$$d\tau^2 = e^{-\frac{2k}{\tau}}dt^2 - e^{\frac{2k}{\tau}}(dR/c)^2. \tag{32}$$

This metric equation represents the weak-field gravitational Lorentz 4-force solution (Eq. 30) under the constraint that it yields a flat spacetime metric for a zero spring constant k or infinite interval τ.

Assume the spring constant k is due to the geometric active mass source, $k = GM/c^2$. Assume the field is static, so $\sigma^2 = R^2 - (ct)^2 \cong R'^2$. Both sigma and tau must have the same magnitude, R. To express the metric in terms of tau real, assume sigma is imaginary, so tau is real:

$$d\tau^2 = e^{-\frac{2GM}{c^2R}}dt^2 - e^{\frac{2GM}{c^2R}}(dR/c)^2. \tag{33}$$

This exponential metric equation has been studied previously.[6, 11, 14, 16] The metric is consistent with weak field tests of gravity to first-order parameterized post-Newtonian accuracy. The exponential metric will be approximately 15% different from the Schwarzschild metric in isotropic coordinates at second-order PPN accuracy. The validity of this unification model can thus be tested experimentally, even if technically quite challenging.

The exponential metric is not a solution to the Einstein field equations. The metric is a singular solution to the GEM field equation that depends only on the Christoffel symbol, Eq. 14. This is a stringent test of consistency of the model.

VII. CLASSICAL GRAVITATIONAL FORCE AND NEW EFFECT

Minkowski spacetime is different from Newtonian space and time due to the way one measures distance, four dimensional versus three. Spacetime symmetry must be broken. The Minkowski interval τ is a consequence of the relationship between time t and space R. The functional relationship between time and space must be severed. In the static field approximation, there is a scalar distance R which is the same magnitude as the interval τ. If the interval τ is replaced by the scalar distance R, then that will sever the functional relationship between time and space:

$$(\frac{dt}{d\tau}, \frac{d\overrightarrow{R}}{cd\tau}) \to (c\frac{dt}{d\mid R\mid}, \frac{d\overrightarrow{R}}{d\mid R\mid}) = (0, \hat{R}). \tag{34}$$

Substitute into weak-field gravitational Lorentz 4-force density equation, Eq. 27 to create a classical 3-force equation:

$$F_g^\mu = (0, -\frac{GM\rho_m}{R^2}\hat{R}) = \frac{d\rho_m U^\mu}{d\tau}. \tag{35}$$

This is not quite Newton's gravitational force density law. The reason is that one must now consider the right-hand side of the force equation carefully. According to the chain rule:

$$\frac{d\rho_m U^\mu}{d\tau} = \rho_m\frac{dU^\mu}{d\tau} + U^\mu\frac{d\rho_m}{d\tau}. \tag{36}$$

An open question is how should spacetime symmetry be broken for the derivatives with respect to the interval τ? An interval is composed of both changes in time and space. For the $\frac{\partial U^\mu}{\partial\tau}$ term, if the interval is only about time, then one gets back Newtonian acceleration, a second derivative of time. For logical consistency, one might be tempted to also substitute time in the $\frac{\partial m}{\partial \tau}$ term. However, the system is presumed to be static, so this would necessarily be zero. If this derivative is to have any chance at being non-zero, it would have to be with respect to the scalar distance R as has been done earlier in the derivation. Note that this new term will not point in a radial direction as gravity has since the time of Newton. Instead, the change in mass in space points in the direction of the velocity of that mass. The classical 3-force law would look like so:

$$-\frac{GM\rho_m}{R^2}(\hat{R}+\hat{V}) = \rho_m \frac{d^2\overrightarrow{R}}{dt^2} + c\overrightarrow{V}\,\frac{d\rho_m}{d\mid R\mid} = . \tag{37}$$

For a point source, the $\frac{d\rho_m}{d|R|}$ term will not make a contribution, and one gets Newton's law of gravity. It is only if the inertial mass is distributed over space as is the case for galaxies or the big bang will the new effect term come into play. If the velocity is constant, then the acceleration is zero. The equation describes the distribution of the inertial mass densityρ_m that contributes to the total gravitational source mass M. The solution to Eq. 37 when there is no acceleration has the inertial mass distribution decay exponentially. There is a problem with the rotation profile of thin disk galaxies.[7, 8] Once the maximum velocity is reached, the velocity stays constant. It has been shown that galaxies should not be stable at all.[13] The new effect term promises a stable decay of the mass distribution for large radii, which sounds like an exact fit. The new effect has an inverse distance dependence, which matches the MOND proposal that has been successfully applied to explain the velocity profile of thin disk galaxies.

Big bang cosmology has two problems: all matter is traveling at exactly the same speed even though it is not possible for them to communicate (the horizon problem), and the model requires high levels of precision on initial conditions to avoid collapse (the flatness problem).[4] Eq. 37 has a stable, constant velocity solution which may resolve both problems of the big bang without the inflation hypothesis. Numerical approaches on Eq. 37 should be conducted.

VIII. QUANTIZATION

The classical electromagnetic Lagrangian cannot be quantized. One way to realize this is to consider the generalized 4-momentum:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial \mathcal{L}_{EM}}{\partial(\frac{\partial A^{\mu}}{c\partial t})} = -F^{\mu 0}. \tag{38}$$

Unfortunately, the energy component of the momentum operator is zero. The commutator of the complementary variables of the potential and energy, $[A^0, \pi^0]$, will equal zero, and cannot be quantized. The momentum for the GEM Lagrangian does not suffer from this problem:

$$\pi^{\mu} = h\sqrt{G}\frac{\partial A^{\mu}}{c\partial t}. \tag{39}$$

When expressed with operators, the commutator $[A^0, \pi^0]$ will not be zero, so the field may be quantized. The field equations (Eq. 5) are similar to the classical electromagnetic Lagrangian with the choice of the Lorenz gauge, the difference being an additional mass current density source. Gupta and Bleuler have quantized the modes of radiation for a four dimensional wave equation.[1, 2] They determined that there were four modes of transmission: two transverse, one scalar, and one longitudinal mode. The scalar polarization mode represent a significant technical problem since its norm may be negative. They introduce a supplemental condition to make the scalar and longitudinal modes virtual.

The field in this model must represent both gravity and electromagnetism. The two transverse modes are spin-1 photons that do all the work of electromagnetism. The symmetric second-rank field strength tensor cannot be represented by a photon because photons transform differently than a symmetric tensor. For a long-range force where like charges attract, the spin must be an even integer. Whatever particle does the work must travel at the speed of light like the transverse modes of transmissions of the field. Since the symmetric tensor is rank two, that can be represented by a spin-2 particle.

The scalar field formed from the trace of $A^{\mu}_{;\nu} + A^{\nu}_{;\mu}$ may be able to play a role similar to the scalar Higgs field. Since the scalar field is entirely a function of the gravity field g^{μ}, the symmetry of the electromagnetic field is not affected. That is a key requirement for any mechanism to introduce mass into the standard model. For the graviton, the trace of the symmetric mixed index tensor must be equal to zero so that the graviton travels at the speed of light.

There are efforts underway to detect the transverse gravitational waves predicted by general relativity. This model predicts the polarization of a gravitational wave will be either scalar or longitudinal, not transverse, because those are the modes of transmission. The detection of the first gravitational wave polarization will mark either success or failure of this unified field theory.

IX. CONCLUSION

Using a nineteenth century approach, an effort to unify physics from the twentieth century has been attempted. A dynamic metric equation is found but it uses a simpler set of field equations than general relativity, a rank one tensor instead of two.

This theory makes several predictions. If weak gravitation effects are measured to second-order parameterized post-Newtonian accuracy, the coefficients for the metric derived here are different from the Schwarzschild metric in isotropic coordinates. It remains to be seen if the complete 3-force equation matches all the data for a thin spiral galaxy or the big bang. The polarization of gravitational waves will be scalar or longitudinal, not transverse as predicted by general relativity. A linear field theory for gravity can work elegantly beside the intellectual superstructure of electromagnetism, using analogy to complete the story of forces in Nature.

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