Gauge Games in a Unified Field Theory

Abstract

Gauge symmetry is enforced in the Maxwell action written with quaternions by a subtraction. The action is rewritten using hypercomplex multiplication rules and the same method to ensure gauge symmetry. The hypercomplex field equations contain Newton's law of gravity, a time dependent term, and an Ampere-like equation. Gauge symmetry for light and gravity can be broken by not doing the subtraction. Yet Maxwell and the hypercomplex field strengths can be combined to form a gauge-invariant unified action. Neither the Higgs mechanism or general relativity are needed.

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When an action *is* invariant under a gauge transformation, it means...

Speed = c

No scalar field

Choose $f(\frac{\partial \phi}{\partial t}, \nabla \cdot A)$

When an action *is not* invariant under a gauge transformation, it means...

Speed = c	Speed < c
No scalar field	Scalar field
Choose $f(\frac{\partial \phi}{\partial t}, \nabla \cdot A)$	Fixed by conditions

The riddle: both are needed in the same action!

Speed = c	Speed < c
No scalar field	Scalar field
Choose $f(\frac{\partial \phi}{\partial t}, \nabla \cdot A)$	Fixed by conditions

Photons + Massive charged particles

The current answer to the riddle is the false vacuum of the Higgs field

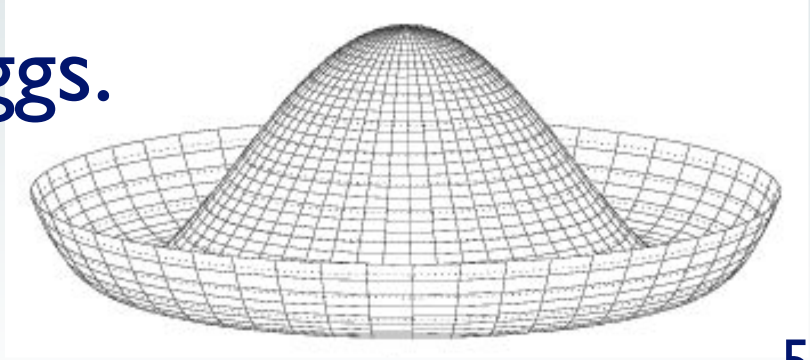
Standard model action has symmetries

U(1), SU(2), and SU(3) [EM, weak , strong]

with *no mass*.

Scalar Higgs field does the Mexican hat *trick*.

LHC's job is to find the Higgs.



Start on path to a unified field theory with the Maxwell action

Antisymmetric rank 2 tensor contractions.

$$S_{\mathrm{EM}} = \int \sqrt{-g}\, d^4x\, (\nabla^\mu A^\nu - \nabla^\nu A^\mu)(\nabla_\mu A_\nu - \nabla_\nu A_\mu)$$

A simple wave equation with U(1) symmetry.

Create variations on Maxwell by rewriting them with quaternions

Use quaternions, a 4D division algebra.

$$S_{\mathrm{EM}} = \int \sqrt{-g}\, d^4x \, (\nabla A - (\nabla A)^*)(A \nabla - (A \nabla)^*)$$

No Greek letters!

Scalar terms identical to tensor equation.

Gauge symmetric because of the subtraction.

Create variations on Maxwell by inserting standard model symmetries

Quaternions can represent the relevant groups.

$$S_{\mathrm{EM}} = \int \sqrt{-g}\, d^4x\, (\nabla A - (\nabla A)^*)(A \nabla - (A \nabla)^*)$$

Weak, electroweak, & strong forces.

$$S_w\!: A \longrightarrow \exp(A - A^*) \qquad [\mathbf{SU(2)}]$$

$$S_{ew}\!: A \longrightarrow \frac{A}{|A|} \exp(A - A^*) \qquad [\mathrm{U(1)}\mathrm{x}\mathbf{SU(2)}]$$

$$S_s\!: A \longrightarrow (\frac{A}{|A|} \exp(A - A^*))^* \frac{B}{|B|} \exp(B - B^*) \quad [\mathbf{SU(3)}]$$

Create variations on Maxwell by using hypercomplex multiplication for gravity

More "symmetric" because $i^2 = +1$.

$$S_G = \int \sqrt{-g}\, d^4x \,(\nabla \boxtimes A^* - (\nabla \boxtimes A^*)^*) \boxtimes (\nabla^* \boxtimes A - (\nabla^* \boxtimes A)^*)$$

Field eqs are like Maxwells, with sign changes.
Example: Gauss' law flip signs as happens
if like charges attract.

Gauge symmetric because of the subtraction.

Subtract Maxwell/gravity actions without the gauge sym. subtractions

EM and gravity densities in the same action.

$$S_{\mathrm{GEM}} = \int \sqrt{-g}\, d^4x \, ((\nabla A)(A \nabla) - (\nabla \boxtimes A^*) \boxtimes (\nabla^* \boxtimes A))$$

Separately, EM and gravity densities are *not* symmetric under a gauge transformation.

Gauge symmetric because of the subtraction.

Predict LHC will not find the Higgs.

Bonus: GEM unified field equations look like Newton's 2nd law

Apply the Euler-Lagrange equation to GEM action to generate the first field equation.

$$\rho = -\frac{\partial^2 A_1}{\partial t\,\partial x_1} - \frac{\partial^2 A_2}{\partial t\,\partial x_2} - \frac{\partial^2 A_3}{\partial t\,\partial x_3}$$

Write in the Lorenz gauge.

$$\rho = \frac{\partial^2 \phi}{\partial t^2}$$

Unification is elegant!