Unifying Gravity and EM by Generalizing EM by sweetser@alum.mit.edu

The EM Lagrange density: $\mathfrak{L}_{\mathrm{EM}} = -\frac{1}{c}\, J_q^\mu\, A_\mu - \frac{1}{4c^2}\, (A^{\mu,\nu} - A^{\nu,\mu})(A_{\mu,\nu} - A_{\nu,\mu})$

Four generalizations:

- Like electric charges **repel** and like mass charges **attract**.
- **A**symmetric field strength tensor tensor, the sum of **anti**symmetric and **symmetric** tensors.
- **Exterior** and **covariant** derivatives.
- **Spin 1** and **spin 2** fields.

The GEM Lagrange density: $\mathfrak{L}_{\mathrm{GEM}} = -\frac{1}{c}\, (J_q^\mu - J_m^\mu)\, A_\mu - \frac{1}{2\,c^2}\, A^\mu{}_{;\nu}\, A_\mu{}^{;\nu}$

Apply the Euler-Lagrange equation to generate the field equations: $J_q^\mu - J_m^\mu = \Box^2 A^\mu$

Generalized Gauss' law: $\rho_q - \rho_m = \nabla_\nu \nabla^\nu \phi = -\vec{\nabla}^2 - \vec{\nabla} \cdot \Gamma^{\sigma i}{}_0\, A_\sigma$ [static law]

1. If $\rho_m = 0$, $\frac{\partial^2 \phi}{c\partial t^2} = 0$, and $\Gamma = 0$, then $\rho_q = -\, c\nabla^2\phi$, which is Gauss' static law.
2. If $\rho_q = 0$, $\frac{\partial^2 \phi}{c\partial t^2} = 0$, and $\Gamma = 0$, then $\rho_m = c\nabla^2\phi$, which is Newton's law of gravity.
3. If $\rho_q = 0$ & $\partial_\nu\, \partial^\nu \phi = 0$, then $\rho_m = \vec{\nabla} \cdot \Gamma^{\sigma i}{}_0\, A_\sigma$, (mass charge density = divergence of the Christoffel).

Exponential metric: $(\partial\tau)^2 = e^{-2\frac{G\,M}{c^2\,R}}\, (\partial\, t)^2 - e^{+2\frac{G\,M}{c^2\,R}}\, (\frac{\partial \vec{R}}{c})^2$

- Same 1st order PPN values as the Schwarzschild metric, so **passes** the same **tests**.
- Different 2nd order PPN values, 0.8 μarcseconds more bending around the Sun (testable, maybe!).
- Solves $\rho_m = \vec{\nabla} \cdot \Gamma^{\sigma i}{}_0\, A_\sigma$! Fun to do on your own :-) [Hint: $A_\sigma = (c/\sqrt{G}, 0, 0, 0)$]
- Linear field, yet metric is consistent with tests of weak and strong equivalence principle.

A 4D linear perturbation near 1/R that is electrically neutral is physically relevant:

diagonal SHO $A^\mu = \frac{c^2}{\sqrt{G}}$

$$\Big(\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} + \frac{k\,c\,t}{\sigma\,2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} + \frac{k\,c\,t}{\sigma\,2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} + \frac{k\,c\,t}{\sigma\,2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} - \frac{k\,c\,t}{\sigma\,2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} + \frac{k\,c\,t}{\sigma\,2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} - \frac{k\,c\,t}{\sigma\,2})^2},$$

$$\frac{1}{(\frac{1}{\sqrt{2}} + \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} + \frac{k\,c\,t}{\sigma\,2})^2} + \frac{1}{(\frac{1}{\sqrt{2}} - \frac{k\,x}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} - \frac{k\,y}{\sigma\,2})^2 + (\frac{1}{\sqrt{2}} + \frac{k\,z}{\sigma\,2})^2 - (\frac{1}{\sqrt{2}} - \frac{k\,c\,t}{\sigma\,2})^2}\Big)$$

The derivative has the correct, classical 1/distance^2 dependence to first order in k:

$$A^\mu{}_{,\nu} \cong \frac{c^2}{\sqrt{G}}\ \frac{k}{\sigma 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Can be used to derive the exponential metric.

A new class of constant velocity solutions for gravity (matches data for NCG3198):

$\vec{F} = -\frac{G\,M\rho}{R^2}\,(\hat{R} + \hat{V}) = \rho\,\frac{V^2}{|R|}\,\hat{R} + \frac{d\rho}{d\,|R/c|}\,\vec{V}$ For constant V: $m = k\,\mathrm{Exp}\,(\frac{G\,M}{c\,V\,R})$

Quantize with 2 spin fields: transverse modes for EM, scalar and longitudinal for gravity.

Conjugate momentum is non-zero: $\pi^\mu = h\sqrt{G}\,\frac{\partial \mathfrak{L}}{\partial \frac{\partial A^\mu}{c\,\partial t}} = h\sqrt{G}\,(-\frac{\partial\phi}{c\partial t}, \frac{\partial A_x}{c\partial t}, \frac{\partial A_y}{c\partial t}, \frac{\partial A_z}{c\partial t})$